Trisect a Line Segment (3)
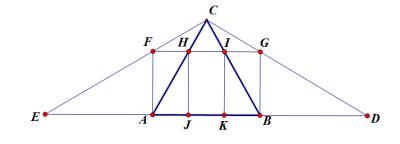
Method:

1) Let's start with the GSP tool for equilateral triangle and create an equilateral triangle ABC
2) Construct collinear segments BD and AE such that BD = AB= AE
3) Construct segments DC and EC
4) Construct perpendicular lines AF and BG from segment AB and points A and B respectively
5) Construct segment FG
6) Segment FG intersects segment AC at point H and segment BC at point I respectively
7) Construct segments HJ and IK, both of which are perpendicular to segments FG and AB.

Claim: Thus points J and K trisect the segment AB.

Proof:

We started with equilateral triangle ABC. So $\angle FCH = 60°$. Since AB = AE = BD, because of equilateral triangle ABC, it implies that AE = AC and BD = BC.

Next, since $\angle CAB = 60° \Longrightarrow \angle CAE = 120°$

Hence $\angle E = \angle FCH = 30°$. Similarly we can derive that $\angle D = \angle GCI = 30°$.

So ΔACE and ΔHCF are similar triangles. Now, since AE = AC we get HF = HC.

Similarly we can show that GI = IC.

Additionally, ΔCHI and ΔABC are similar. So CH = IC.

So we can establish that GI = IC = IH = HC = HF

Hence, GI = IH = HF, and it implies, AJ = JK = KB.

So points J and K trisect the segment AB.